Fatigue crack propagation after overloading and underloading at negative stress ratios

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Abstract

This paper intends to evaluate the influence of the intrinsic properties of the materials, namely plastic and cyclic plastic properties, on the overloading/underloading effect on crack propagation rate, at baseline negative stress ratios, under plane strain conditions. The importance of the negative loading part of the fatigue cycle on crack propagation rate has been shown by previous works of this same author. In those works has also been shown that under baseline negative stress ratios there exists negative open loads and crack propagation rate does not correlate properly with the crack closure concept. These features were shown to be strongly related to plastic properties and cyclic plastic properties of the materials. It has been concluded that the Bauschinger effect may be the explanation for the different sensitivity to negative loads. Thus, some materials may be very sensitive to negative loads and some others may not be so sensitive. Tensile overload and compression underload tests, at positive and negative baseline stress ratios were made in different materials, with different plastic properties, in order to predict their influence on crack propagation rate. The main emphasis in this paper is the importance of the compressive part of the loading cycle under negative baseline R ratios on overloads/underloads effect on crack propagation rate. Results will show that the effect of overloads and underloads on crack propagation rate, at baseline negative stress ratios, are not fully accounted for by crack propagation models and that the generalized accepted behaviour of OL/UL may not be the same at baseline negative stress ratios. It will be shown that Overloads may produce acceleration instead of the accepted retardation effect. A physical understanding on the effects of OL/UL is also provided in the paper.

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Keywords: Fatigue crack growth; Overload; Underload; Negative stress ratios; Bauschinger effect

1. Introduction

Overloads/underloads occur very frequently under baseline negative stress ratios (it is a common load condition on several components such as on almost all rotating components (camshafts, crankshafts, etc.)). However, there are very few papers of the effect of overloads and underloads (compression overloads) at negative stress ratios. The reason maybe that the compressive load part of fatigue traditionally not taken into consideration on fatigue analysis. It is not included in the calculation of the stress intensity factor range, $\Delta K$. The ASTM recommendation E 647-95a is that for negative stress ratios, the negative loading part of the cycle should be neglected. Thus, at negative stress ratios, $\Delta K$ is considered equal to $K_{\text{max}}$. As a fact, it is assumed that when the load is negative the crack is closed and there is no growth of the crack when it is not open. However, different authors over the last decades highlighted that the negative portion of the loading cycle (at negative stress ratios) may have a strong influence on crack growth. This effect was verified in two different situations:

1.1. Fully compressive tests

Fleck [1], Suresh [2], Pippan [3], in the eighties, and more recently, Hermann [4] and Kasaba [5] reported the
nucleation and growth of cracks in specimens under fully compressive loadings. These cracks may grow with a decreasing growth rate until complete crack arrest occurs at a certain crack length.

1.2. Tensile-compression tests

It has been reported during the last decades that the compression part of the loading, under tensile-compression tests (negative $R$ ratios), may have a substantial influence on the behavior of long cracks [6–14]. The different previous authors [6–14] used the closure phenomena, either roughness induced closure and plastic induced closure, to explain this behaviour.

Pommier et al. [10], on a study with a N18 base superalloy evidenced the strong detrimental effect of the compressive part of the cycle. Recently, Pommier [11], also showed how a compressive load can be detrimental in certain materials. The crack growth rate increased by a factor of five in a test with a negative load part as compared to a test without negative loading, for the same $K_{\text{max}}$. This behaviour was attributed to plastic properties of the material, and in particular the kinematic hardening of the alloys.

Previous papers of the author of this paper, namely [12,14] have shown the importance of the negative loading part of the cycle on crack propagation rate. It has been shown some important features related to negative stress ratios that strongly affect crack growth, namely:

- there may exist negative open loads for negative stress ratios;
- crack propagation does not correlate properly with the crack closure concept;
- crack propagation may be strongly affected by negative external loads.

These features were reported to be strongly related to the plastic properties and cyclic plastic properties of the materials. Some materials may have negative open loads or positive ones, at the same negative stress ratio, depending on the Bauschinger effect. The crack propagation rate, for the same negative $R$ ratio, may also strongly change with the minimum load (negative), depending also on the cyclic plasticity properties of the materials.

1.3. Effect of overloads and underloads (compressive overloads)

Based on the previous studies [12,14] it is reasonable to expect that, if the negative loading part of the cycle affect crack initiation and crack propagation in constant amplitude loading – CAL, it may also have some influence on variable amplitude loading, VAL.

Transient effects such as overloads and underloads perturb steady state fatigue crack growth and affect the growth rates by retarding or accelerating the growth. Quantification of these effects has been the subject of intensive study for more than 30 years. It is widely accepted that overloads produce retardation and underloads produce acceleration, and combined overload–underloads have mixed effects depending on the sequence. However, the dominant mechanisms involved are not consensual among the fatigue community. Furthermore it must be said that almost all performed tests found in published research papers were under positive $R$ ratios. The effect of overloads and underloads under negative stress ratios have not been extensively investigated.

It is interesting to observe that at negative stress ratios, the behaviour of overloads and underloads may not be the one generally accepted by the fatigue research community. While Dabayeh [15] reported the same accepted behaviour for positive and negative stress ratios, Halliday [16], reported that overloads do not have any influence on crack growth at $R = -1$ while it has a retardation effect at $R = 0.05$.

Makabe and McEvily [17] recently reported that in the case of a negative baseline stress ratio, fatigue crack growth rate can actually accelerate after a tensile overload instead of having a retardation effect, as widely accepted. In this last case, the acceleration effect of an overload was attributed to the small thickness of the specimen (plane stress condition) giving origin to surface deformation. Notwithstanding Makabe’s explanation is not contradictory with the overall accepted behaviour (retardation after overloads and acceleration after underloads) his result shows that, maybe, the effect of overloads and underloads, on crack propagation rate, at negative stress ratios, is not fully understood.

It will be verified in this paper that at baseline negative stress ratios there are particular and important features that become dominant with the presence of negative loads. And it will be verified that at negative stress ratios overloads and underloads may have a much different effect then they have at positive stress ratios. It will be shown that an overload may cause acceleration even under plain strain conditions, at negative stress ratios. That effect is related to cyclic plastic properties of the material and not to surface deformation of thin specimens under plane stress conditions as reported by Makabe and McEvily [17].

The focus of this paper is on the effect of single overloads and underloads on CAL at baseline negative stress ratios.

It is also the intention of this paper to correlate the intrinsic properties of the materials, namely plastic and cyclic plastic properties (including the Bauschinger effect), to the overloading/underloading effect on crack propagation rate, under plane strain conditions.

This paper will also provide a physical explanation based on two competing mechanisms: damage accumulation and residual stress shielding to explain the behaviour of crack growth after Overloads/underloads.

2. Experimental methods and materials

2.1. Materials

Materials used in this work were chosen in order to cover a wide range of cyclic plastic properties and fatigue
crack surface characteristics, namely: a low carbon steel (Ck45 – cyclic hardening and high Bauschinger effect; Ti6Al4V – cyclic softening and low Bauschinger effect, and aluminum, Al 7175 – cyclically neutral and low Bauschinger effect). Generally, aluminium alloys and titanium alloys are considered to be sensitive to roughness induced crack closure (RICC) while steels are more dependent on plastic properties. Mechanical properties and monotonic and cyclic properties are listed on Table 1.

The BAU (Bauschinger) effect was calculated with Eq. (1). One cyclic stress–strain curve can be seen in Fig. 1 from where values of $S_{ys}$ and $\sigma_{ys-rev}$ were obtained (see also Table 2).

$$BAU = \left[ (S_{ys} - \sigma_{ys-rev})/\sigma_{ys} \right]$$

(1)

2.2. Test procedures

Fatigue tests were performed on a Dartec servo-hydraulic testing machine under a two baseline $R$ ratios: $R = 0$ and a negative stress ratio, $R = -1$. Tests were conducted in laboratory air, 20 $^\circ$C, using a sinusoidal loading, under loading control at a frequency of 2 Hz. Tests were stop, the overloads/underloads were manually applied, and tests were start at the same frequency.

Overloads and underloads were applied for crack lengths of approx. 1.5 to 2.5 mm in depth.

A Pulsed DCPD system was used to measure crack growth.

For each material different stress conditions were used. Fig. 2a and b, illustrate the stress conditions. On Table 3, their values are registered.

2.3. Specimens

Notched solid round specimens, ø12 mm, according to ASTM E 606-80 were used in this investigation [12,14].

2.4. Crack closure measurement

In this study, the near tip strain gauge method is used for the crack closure measurement. This technique has been traditionally used for pure Mode I crack closure measurements [18]. A strain gauge, with $1 \times 1.5$ mm size was glued on the specimen surface. The strain gauges are located 2 mm above or below the crack, in order to avoid the free surface effects in the measurements. Tests were performed at ambient temperature. Strain values were recorded with

Table 1

<table>
<thead>
<tr>
<th>Mech. Propert/Material</th>
<th>Ti6Al4V</th>
<th>Al7175</th>
<th>Ck 45</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{ys(0.2%)}$ (MPa)</td>
<td>989</td>
<td>461</td>
<td>499</td>
</tr>
<tr>
<td>$\sigma_{uts}$ (MPa)</td>
<td>1055</td>
<td>535</td>
<td>793</td>
</tr>
<tr>
<td>$E$ (MPa)</td>
<td>$1.15 \times 10^5$</td>
<td>$0.74 \times 10^5$</td>
<td>$2.00 \times 10^5$</td>
</tr>
<tr>
<td>$\varepsilon_y$ (%)</td>
<td>16.1</td>
<td>13.8</td>
<td>23.5</td>
</tr>
<tr>
<td>$n$</td>
<td>0.1067</td>
<td>0.1177</td>
<td>0.2452</td>
</tr>
<tr>
<td>$k$</td>
<td>1733.1</td>
<td>768.4</td>
<td>1551.8</td>
</tr>
<tr>
<td>$n'$</td>
<td>0.0650</td>
<td>0.0870</td>
<td>0.2340</td>
</tr>
<tr>
<td>$k'$</td>
<td>1185.0</td>
<td>740.9</td>
<td>1740.8</td>
</tr>
<tr>
<td>Grain size ($\mu$m)</td>
<td>12</td>
<td>4.5</td>
<td>35</td>
</tr>
</tbody>
</table>

$n$, monotonic hardening exponent; $k$, monotonic hardening coefficient; $n'$, cyclic hardening exponent; $k'$, cyclic hardening coefficient.

Table 2

<table>
<thead>
<tr>
<th>Bauschinger effect parameter</th>
<th>Ti6Al4V</th>
<th>Al 7175</th>
<th>Ck45</th>
</tr>
</thead>
<tbody>
<tr>
<td>BAU</td>
<td>1.51</td>
<td>1.67</td>
<td>1.18</td>
</tr>
</tbody>
</table>

Values obtained from curves on Ref. [12].

$BAU = \left[ (S_{ys} - \sigma_{ys-rev})/\sigma_{ys} \right] - $ Eq. (1).
a HBM MGCplus equipment. Measurements of crack opening load were recorded at different crack length before and after the overloads and underloads. More details about the gauge set up and measurement may be found on Ref. [14].

3. Results

Figs. 3–14 present the results for the three materials reported in this paper, for both overloads and underloads, and for \( R = 0 \) and \( R = -1 \). All Figs show: (a) graphs a versus number of cycles, (b) \( da/dN \) versus \( D_K \) and (c) \( K_{op} \) versus (a).

Results of graphs from Figs. 3–14 can be summarized as follows:

3.1. Acceleration/retardation after underloads/overloads

- At negative loading cycle \( (R = -1) \) an underload produces nearly no effect on Al7175 and Ti6Al4V (Figs. 3 and 7), and acceleration on Ck45 (Figs. 11);
- At negative loading cycle \( (R = -1) \) an overload produces nearly no effect on Al7175 and Ti6Al4V (Figs. 4 and 8), and acceleration on Ck45 (Fig. 12);
- At a positive loading cycle \( (R = 0) \) an underload produces acceleration on Al7175 and Ti6Al4V (Figs. 5 and 9), and has nearly no effect on Ck45 (Fig. 13);
- At a positive loading cycle \( (R = 0) \) an overload produces retardation on Al7175 and Ti6Al4V (Figs. 6 and 10), and has nearly no effect on Ck45 (Fig. 14);

3.2. Crack closure/crack growth

- In almost all cases there is a good qualitative agreement between crack growth changes and crack opening values, e.g. a decrease in crack opening values causes a crack acceleration and vice-versa (Figs. 3c–14c).

<table>
<thead>
<tr>
<th>Test Parameter</th>
<th>Ti6Al4V</th>
<th>Al 7175</th>
<th>Ck45</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant amplitude ( R = 0 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( K_{max} ) (MPa ( \cdot ) m(^{1/2}))</td>
<td>14.59</td>
<td>9.06</td>
<td>13.29</td>
</tr>
<tr>
<td>( K_{min} ) (MPa ( \cdot ) m(^{1/2}))</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( K_{OL} ) (MPa ( \cdot ) m(^{1/2}))</td>
<td>21.65</td>
<td>16.17</td>
<td>19.91</td>
</tr>
<tr>
<td>Overload ratio – OLR</td>
<td>1.5</td>
<td>1.8</td>
<td>1.5</td>
</tr>
<tr>
<td>( K_{max} ) (MPa ( \cdot ) m(^{1/2}))</td>
<td>13.34</td>
<td>6.79</td>
<td>12.55</td>
</tr>
<tr>
<td>( K_{min} ) (MPa ( \cdot ) m(^{1/2}))</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( K_{UL} ) (MPa ( \cdot ) m(^{1/2}))</td>
<td>-20.00</td>
<td>-12.12</td>
<td>-18.80</td>
</tr>
<tr>
<td>Underload ratio – ULR</td>
<td>-1.5</td>
<td>-1.8</td>
<td>-1.5</td>
</tr>
<tr>
<td>Constant amplitude ( R = -1 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( K_{max} ) (MPa ( \cdot ) m(^{1/2}))</td>
<td>11.52</td>
<td>6.34</td>
<td>11.84</td>
</tr>
<tr>
<td>( K_{min} ) (MPa ( \cdot ) m(^{1/2}))</td>
<td>-11.52</td>
<td>-6.34</td>
<td>-11.84</td>
</tr>
<tr>
<td>( K_{OL} ) (MPa ( \cdot ) m(^{1/2}))</td>
<td>17.27</td>
<td>11.32</td>
<td>17.74</td>
</tr>
<tr>
<td>Overload ratio – OLR</td>
<td>1.25</td>
<td>1.4</td>
<td>1.25</td>
</tr>
<tr>
<td>( K_{max} ) (MPa ( \cdot ) m(^{1/2}))</td>
<td>14.08</td>
<td>7.36</td>
<td>3.25</td>
</tr>
<tr>
<td>( K_{min} ) (MPa ( \cdot ) m(^{1/2}))</td>
<td>-14.08</td>
<td>-7.36</td>
<td>-13.25</td>
</tr>
<tr>
<td>( K_{UL} ) (MPa ( \cdot ) m(^{1/2}))</td>
<td>-22.18</td>
<td>-13.15</td>
<td>-19.85</td>
</tr>
<tr>
<td>Underload ratio –ULR</td>
<td>-0.25</td>
<td>-0.4</td>
<td>-0.25z</td>
</tr>
</tbody>
</table>

\[
OLR/ULR = \frac{K_{OL} - K_{min}}{K_{max} - K_{min}}
\]

In this equation \( K_{OL} \) for ULR is the minimum \( K \) of the underload cycle.
4. Discussion

The discussion will be made taking into consideration the following analysis:

A. Brief review of existing mechanisms based on existing explanations and correlation with results in this study;

B. Explanation of results based on a competition between a damage accumulation effect and a shielding effect.

4.1. Brief review of existing mechanisms based on existing explanations and correlation with results in this study

A very brief review of the overloads/underloads mechanisms that contribute for retardation/acceleration and that
could be related to the present paper will be consecutively made. This short review is based on Refs. [19,20]

(a) residual stress effects;
(b) crack tip blunting;
(c) crack front irregularities;
(d) plasticity induced crack closure;
(e) roughness induced closure;
(f) influence of thickness;
(g) strain hardening.

4.1.1. Residual stress effects

This theory assumes that after an OL compressive residual stresses are created in a small region ahead of

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Fig. 6. Fatigue crack growth data for Al7175, R = 0. Overload: (a) crack length-number of stress cycles; (b) da/dN-ΔK and (c) Crack closure (Kop)-crack length.

Fig. 7. Fatigue crack growth data for Ti6Al4V, R = −1. Underload: (a) crack length-number of stress cycles; (b) da/dN-ΔK and (c) Crack closure (Kop)-crack length.
the crack tip (the reference state of the external load at which the residual stress field is specified is zero load). This compressive stress field behaves like a shielding effect to crack propagation. After an UL is a tensile residual stress field that is created at the crack tip. Thus, there must exist retardation after an OL and acceleration after an UL. This theory is based on the monotonic plastic zone area created after an OL/UL. This theory is not able to explain why after an overload there exists acceleration instead of retardation as presented in this paper. Thus, it is considered that this theory, although able to explain very well almost all OL/UL situations and physically reasonable is not able, alone, to explain the results of this paper.

Fig. 8. Fatigue crack growth data for Ti6Al4V, $R = -1$, Overload: (a) crack length-number of stress cycles; (b) $da/dN$-$\Delta K$ and (c) Crack closure ($K_{op}$)-crack length.

Fig. 9. Fatigue crack growth data for Ti6Al4V, $R = 0$, Underload: (a) crack length-number of stress cycles; (b) $da/dN$-$\Delta K$ and (c) Crack closure ($K_{op}$)-crack length.
4.1.2. Crack tip blunting

This explanation assumes that after an overload the tip blunts. The retardation is the number of cycles required to reinitiate and propagate the crack from this notch. This theory is not also able to explain why there is acceleration after an OL as presented on this study.

4.1.3. Crack front irregularities

This theory assumes that after an OL/UL crack front irregularities, crack branching and crack deflection can be generated. These features can affect the local stress intensity factor. Although, it is clear that OL/UL may affect crack front topography it is difficult to accept that when changing from \( R = 0 \) to \( R = -1 \), in some materials the effect of an OL keeps the same (for example retardation) and in other mate-

**Fig. 10.** Fatigue crack growth data for Ti6Al4V, \( R = 0 \), Overload: (a) crack length-number of stress cycles; (b) \( \frac{da}{dn} \)-\( \Delta K \) and (c) Crack closure (\( K_{op} \))-crack length.

**Fig. 11.** Fatigue crack growth data for Ck45, \( R = -1 \), Underload: (a) crack length-number of stress cycles; (b) \( \frac{da}{dn} \)-\( \Delta K \) and (c) Crack closure (\( K_{op} \))-crack length.
rials (for example Ck45) the behaviour changes from retardation to acceleration, as happens in this work. Thus, this theory will not be considered in this paper.

4.1.4. Plasticity induced crack closure

Plasticity induced crack closure has been used to explain the three stages of an OL: the brief initial acceleration can be attributed to a reduced contact between crack flanks (reduced Pop); as the crack advances a wedge of the stretched material is formed between the crack faces which results in a gradual increase in Pop and crack retardation; as the crack grows to outside the plastic deformation area the stretched material influence decays to nearly no effect (return to a prior overload Pop). The plasticity induced crack closure theory is also able to explain the effect of UL. However, recent studies [12,14] have shown that the crack closure theory does not.

Fig. 12. Fatigue crack growth data for Ck45, $R = -1$, Overload: (a) crack length-number of stress cycles; (b) $da/dN$-$\Delta K$ and (c) Crack closure ($K_{op}$)-crack length.

Fig. 13. Fatigue crack growth data for Ck45, $R = 0$, Underload: (a) crack length-number of stress cycles; (b) $da/dN$-$\Delta K$ and (c) Crack closure ($K_{op}$)-crack length.
apply at negative stress ratios. Furthermore, although it is able to explain most features of fatigue crack growth, it should be understood more as a consequence than as the reason for the fatigue behaviour.

4.1.5. Roughness induced closure

Roughness induced crack closure is also able to produce a delayed retardation after an OL or acceleration after an UL. However, it has been shown in some papers [12,14,21] that roughness does not play an important role on crack propagation at negative stress ratios. Plasticity is dominant under negative baseline stress ratios. On the other hand roughness induced closure is not able to explain why after an OL acceleration may occur. It only explains retardation after an OL. Thus, it does not seem that roughness may explain the results obtained in this paper.

4.1.6. Influence of thickness

Thickness influence is related to the plane stress or plane strain condition. Accentuated effects are expected for thinner specimens (plane stress condition). However, in both conditions it is expected that an OL produces retardation. Makabe and McEvily [17] reported crack acceleration after an OL at $R = -1$. They attributed the acceleration effect to the thickness effect and to plastic deformation at the overload zone. They said that, after the OL, upon unloading from the tensile load level to the compressive loading level (at $R = -1.5$) the plastic deformed material would be forced outward, and instead of a contraction, a bulge would start to develop. Consequently when returning to zero load the surface overload region will be in a state of residual tension rather than compression, and the crack would accelerate rather than being retarded. However, this explanation (the thickness effect) is not able to explain acceleration under plain strain conditions.

4.1.7. Strain hardening

Yield stress, strain hardening and other mechanical properties have been suggested to affect the OL/UL effect on different materials [19,20]. This reasoning is absolutely true: materials behaviour must be affected by its inherent properties. The property that is commonly referred as the one that most affect crack growth after OL/UL is monotonic strain hardening. More strain hardening materials exhibit more pronounced retardation than a less strain hardening material. However, it is not explained how strain hardening would be able to shift the retardation behaviour to an acceleration one after an overload. Thus, it seems that strain hardening is not useful, by itself, to explain crack acceleration after an overload, as presented in the present paper. However, it will be shown that crack growth behaviour after OL/UL may have an explanation, including other intrinsic material properties.

None of the existing theories, above presented, seems to be able to explain acceleration after an overload. The following point of this paper will present an explanation for this behaviour based on a competition between two mechanisms: damage accumulation and residual stress effect.

4.2. Competition between a damage accumulation effect and a residual stress effect

With exception of one case the results presented in this paper follow a general tendency of the accepted OL/UL behaviour. They will be briefly presented.
Overloads have a general trend to cause retardation on fatigue crack growth. This effect is dependent on material and OLR. This is what was obtained in this paper for all alloys (Figs. 6 and 10). An exception occurred for Ck45 alloy at negative $R$ ratios (Fig. 12). This behaviour will be discussed later. When OLR is low it is possible that the overload effect is not significant (Figs. 4, 8, 14).

Underloads have a general trend to cause an acceleration of fatigue crack growth. This effect is also dependent on material and OLR. This is what was obtained in this paper for all alloys (Figs. 5, 9, 11). Again due to low underload ratios, in some cases there were no substantial effect (Figs. 3, 7, 13).

The effect of $R$ ratio seems to be material sensitive as well. Some materials increase its sensitiveness to OL/UL for positive $R$ ratios, like Al and Ti and other materials are more sensitive to OL/UL at negative $R$ ratios, like Ck45 alloy.

The sensitiveness to OL and UL seems to be also material dependent. Some materials seem to be more sensitive to ULs (ck alloy) and other materials to OLs (Al and Ti).

Relating to the apparently strange behaviour observed for Ck45 alloy, at negative stress ratios, e.g. crack acceleration after an OL, a physical explanation for that behaviour will be proposed. It will be verified that the explanation is also adequate for all other conditions namely different OL ratios, negative and positive stress ratios, different material properties, transient effects such as immediate crack acceleration after OL and before retardation, etc.

The crack acceleration after the OL may be explained based on two competing mechanisms: residual stress effect and fatigue damage accumulation. These mechanisms act at the crack tip and depend on the monotonic plastic zone size (MPZS) and the monotonic properties (residual stress effect) and on the cyclic plastic zone size (CPZS) and cyclic properties of the different materials (damage accumulation effect). It is widely accepted that there are two plastic zone sizes at the crack tip. Bathias [22] detected and measured a reversed plastic zone within the monotonic plastic zone. Thus, there are two possible plastic effects at the crack tip. Nicoletto [23], using Moiré interferometric observations of plastic zone sizes verified that during an overload the monotonic plastic zone size increases. He also observed that the cyclic plastic zone size increases for a brief period and then decreases for a size smaller that the one previous to the OL. After a certain crack propagation length, following overload, both monotonic and cyclic zone sizes become similar to the ones before the overload. This effect can be schematically illustrated in Fig. 15. Nicoletto [23] also showed that this behaviour is properly predicted by the Fuhring–Seeger plasticity model.

The first aspect to highlight is that during an overload (either in tension or in compression) both the monotonic and the cyclic plastic zone sizes are affected. The second aspect to highlight is that, at positive stress ratios the difference between the CPZS and MPZS is big but as the $R$ ratio decreases, which happens for negative stress ratios, the difference between the CPZS and MPZS also decreases and, at $R = -1$ they have almost the same size.

Rice [24] defined expressions for a monotonic plastic zone and, within that, for a cyclic or reversed plastic zone. As follows:

$$r_p^m = \frac{1}{\pi} \left( \frac{K_{\text{max}}}{\sigma_{\text{ys}}} \right)^2$$  \hspace{1cm} (2)

$$r_p^c = \frac{1}{\pi} \left( \frac{\Delta K_{\text{max}}}{2\sigma_{\text{ys}}} \right)^2$$  \hspace{1cm} (3)

Eqs. (2) and (3) are described in Fig. 16 for a certain $K_{\text{max}}$. They show that as the stress ratio $R$ decreases and for the same $K_{\text{max}}$, the cyclic plastic zone size increases. The cyclic plastic zone size reaches the size of monotonic plastic zone size when $\Delta K_{\text{max}} = 2 \times K_{\text{max}}$. This is what happens for $R = -1$. These analyses are for plane stress. For plane strain, plastic zone, sizes are estimated to be smaller by a factor of three. Later, McClung [25] specified that $\sigma_{\text{ys}}$ in Eq. (3) should be replaced by the cyclic yield strength, $\sigma_{\text{cys}}$, so that the size of the reversed plastic zone is influ-
enced by cyclic hardening or softening. As the difference between CPZS and MPZS decreases for negative stress ratios it is expected that cyclic plasticity effect on OL/ULs becomes more important at negative stress ratios than at positive stress ratios. And it is also reasonable to accept that the effect based on the monotonic plastic zone size decreases for negative stress ratios, as the cyclic plastic effect increases.

It is assumed herein that the monotonic zone size is related to the residual stress effect. After an overload the monotonic plastic zone area will promote a shielding effect because strong compressive residual stresses arise in that area [26,27]. It is also accepted that the cyclic zone area is related to the damaging effect. Damage accumulation occurs in that area [28–31].

An analysis on the overload described by Nicolletto [23] in Fig. 15 shows that after the overload both the monotonic and the cyclic plastic areas increase. As a consequence of the huge increase of MPZS a big shielding effect to crack propagation is created within the monotonic area (see Fig. 17). However, the cyclic plastic area also grows increasing the accumulation of damage. This accumulation in damage quickly disappears due to the compressive effect of the monotonic shielding big area. It is interesting to verify that based on the increase in accumulation immediately after the overload it is possible to explain the immediate acceleration just after the overload. A bigger accumulation of damage means faster crack propagation rates. But the huge shielding effect of the compressive field around the cyclic plastic area causes the decrease of the CPZS and crack propagation rate starts to reduce to a lower value (smaller size of cyclic zone size) then before the overload. This explains the overall crack retardation observed after the OL. After the crack surpasses the overload affect region both plastic areas return to their normal size.

Based on the same reasoning the effect of an underload can be schematically shown in Fig. 18. After an underload both the monotonic and the cyclic plastic areas increase. Now, the big monotonic deformed area acts not as a shield for crack propagation but as a facilitator because that area is under a tensile residual stress field. This tensile residual stress field will allow the cyclic plastic area to increase while

\[ R \geq 0 \]

\[ \text{cyclic} \]

\[ \text{monotonic} \]

\[ \text{OL} \]

\[ \text{UL} \]

\[ \text{Crack length} \]

\[ \text{Plastic Zone Size} \]

(a) (b) (c)

Fig. 17. Schematic effect of overload on both monotonic and cyclic plastic zone sizes at positive stress ratios: (a) before OL; (b) immediately after OL and (c) after OL.

Fig. 18. Schematic effect of underload on both monotonic and cyclic plastic zone sizes at positive stress ratios: (a) before UL; (b) immediately after UL and (c) after UL.
the CPZ is inside the monotonic UL affected region. Thus, there is acceleration immediately after the underload and also during the crack growth inside the underload affected region. It is interesting to note that, based on these assumptions it is not possible that after an underload there exists an immediate retardation followed by acceleration. And it is interesting to note that it is not found in the literature this transient retardation after an UL. Thus, both the monotonic area (tensile residual stresses) and the cyclic plastic area (damage accumulation) induce faster crack propagation rates. Furthermore, as both mechanisms are contributing to crack acceleration it would be reasonable to say that materials should be more sensitive to ULs then to OLs.

Apparently none of the previous explanations would be able to explain the unexpected accelerating behaviour of an overload at $R = -1$. But this is not the case. It will be shown that the explanation is also adequate for crack acceleration after an overload.

Under negative stress ratios, the cyclic plastic zone has almost the same size of the monotonic one, as presented in Fig. 16 and Eqs. (2) and (3). Thus, the effect of an overload could be explained based in Fig. 19. After an overload both the monotonic plastic area and the cyclic plastic areas increase. Due to increase in CPZS (damage accumulation) immediate acceleration after the OL is expected. And, as the cyclic zone size has about the same size of the monotonic one, the residual stress shielding effect (compressive residual stresses) is now much smaller then it was at positive stress ratios (see Figs. 17 and 20). Thus, although it will affect the cyclic plastic area by reducing it, its effect is now small. Thus, it is possible that during the OL affected region the CPZS would always remain bigger then its size previous to the overload. This means that the damage accumulation effect would then be bigger then the residual stress shielding effect. As a consequence the overall behaviour after the overload would be acceleration instead of retardation. This competing mechanism would then be able to explain crack acceleration after an OL for certain loading conditions (negative baseline stress ratios).

Accepting this possible competing mechanism the question that arises is why this acceleration effect occurs for one material (Ck45) and does not occur for other materials (Ti and Al)? The answer for this question may be found on cyclic plastic properties of materials and in particular on Bauschinger effect. Pommier [11] has shown by doing numerical simulations that the effect of a high Bauschinger effect is a spreading of the cyclic plastic zone ahead of the crack tip. As a fact the highest the Bauschinger effect on a material the largest the reverse plasticity at the crack tip during unloading. The result is a higher cyclic damaged area at the crack tip.

When comparing the three alloys used in this study it can be seen (Table 2) that the Ck45 alloy displays a high Bauschinger effect while the other two alloys display a low Bauschinger effect. Thus, it is expected that the CPZS ahead of crack tip is much bigger for materials with high Bauschinger effect. This means that the damaging effect,
Arrived to this point it is interesting to verify that the relationship crack propagation to Bauschinger effect, found in this paper for VAL – variable amplitude loading, in particular for OL/UL was also verified for constant \( R \) ratio. It was found again at negative stress ratios, e.g. when the importance of cyclic plasticity increases as compared to monotonic plasticity. As a fact, in [14] was found that at \( R = -1 \) the effect of the negative part of the load cycle was very detrimental for fatigue crack propagation only for materials with high Bauschinger effect. It was also verified in that paper that the other materials (low Bauschinger affected materials) remained almost insensitive to the negative loading part of the cycle. Furthermore, it was concluded in [14] that damage accumulation principles along with residual stresses concepts seemed to be adequate to explain constant amplitude crack propagation. On VAL, Romeiro [32] also correlated the Bauschinger effect with crack propagation for different number of consecutive OLs and ULs.

Based on this reasoning it is expected that as CPZS increases (\( R \) ratio decreases) the Bauschinger sensitive materials (\( R \) ratio decreases) become more sensitive to OL/UL while the Bauschinger insensitive materials become less sensitive to OL/UL. This is exactly what is found on this paper results: as \( R \) ratio decreases Ck45 alloy (Bauschinger sensitive) becomes more sensitive to OL/UL. The other two alloys become less sensitive.

Notwithstanding all plastic properties involved in the process, monotonic and cyclic properties, kinematic and isotropic hardening, as well as the Bauschinger effect, would need to be taken into consideration in order to fully quantify the OL/UL effects, it seems that at negative stress ratios, the Bauschinger effect seems to be significantly important so that the fatigue propagation results both in CAL and VAL match very well with the Bauschinger effect.

5. Conclusions

The main conclusions of this work are the following:

- The negative baseline loading part of the cycle plays a significant role on the effect of overloads and underloads;
- At negative stress ratios an overload may cause an acceleration of fatigue crack propagation instead of the expected retardation;
- Fatigue crack growth at negative stress ratios seems to be controlled by cyclic plasticity and cyclic plastic properties of the materials;
- The Bauschinger effect seems to be strongly related to the OL/UL response of materials at negative stress ratios;
- A competing mechanism between damage accumulation and residual stress shielding effect seems to be adequate to explain the effect of overloads and underloads both for negative and positive stress ratios.

References


Fig. 22. Schematic comparison of the Bauschinger effect on competing mechanisms at \( R = -1 \).